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The Number of States of Two Dimensional Critical String Theory

Tom Banks

Department of Physics and Astronomy

Rutgers University, Piscataway, NJ 08855-0849

Leonard Susskind

Physics Department

Stanford University, Stanford, CA 94305

We discuss string theory vacua which have the wrong number of spacetime dimensions, and give a crude argument that vacua with more than four large dimensions are improbable. We then turn to two dimensional vacua, which naively appear to violate Bekenstein's entropy principle. A classical analysis shows that the naive perturbative counting of states is unjustified. All excited states of the system have strong coupling singularities which prevent us from concluding that they really exist. A speculative interpretation of the classical solutions suggests only a finite number of states will be found in regions bounded by a finite area. We also argue that the vacuum degeneracy of two dimensional classical string theory is removed in quantum mechanics. The system appears to be in a Kosterlitz-Thouless phase. This leads to the conclusion that it is also improbable to have only two large spacetime dimensions in string theory. However, we note that, unlike our argument for high dimensions, our conclusions about the ground state have neglected two dimensional quantum gravitational effects, and are at best incomplete.

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1. Introduction

String theory appears to have many classical ground states which bear no resemblance whatsoever to the real world. In particular, many of them have the wrong number of large spacetime dimensions. What is perhaps more disturbing, is that many of these wrong dimensional vacua have a high degree of supersymmetry. Very strong symmetry arguments then suggest that they are exact, stable quantum ground states of the system. Why then are they not realized in the world of phenomena?

For ground states of high spacetime dimension, the beginnings of an argument can be constructed. Imagine, following Einstein, that there is a general principle that only compact spatial dimensions need to be considered. Then the idea of decoherence of states with different expectation values of homogeneous scalar fields is only an approximate one. Instead of a moduli space of vacua we would have a ground state wave function which is a function on moduli space. In a low energy, "minisuperspace" approximation (and neglecting for the moment the coupling to gravity) it would be the ground state of the nonlinear quantum mechanics model on moduli space defined by the tree level string effective action. Apart from gravitational effects, corrections to this approximation can be controlled by constructing an effective action for the homogeneous modes by integrating out string modes with finite mass and momentum. The corrections are negligible near the boundaries of moduli space where more than four dimensions are large. Indeed, in high dimension this is true even for gravitational effects. In particular, infrared instabilities which could lead to a potential on moduli space, and perhaps inflation, vanish in the limit in which the system has more than four noncompact dimensions. When the theory has a sufficiently large supersymmetry, they are completely absent. We conclude that the dynamics of the homogeneous modes is well approximated in this limit by the quantum mechanics of free motion on a noncompact space of finite volume[1]. The ground state wave function is a constant and the probability of being at any particular point of moduli space is determined by the Zamolodchikov volume element. *The probability of radial moduli being much larger than the string scale, vanishes like a power of the radius.*

When we consider the coupling of the system to gravity, this reasoning can be falsified by the phenomenon of inflation. We will not go into a detailed discussion of how quantum

string dynamics in four dimensions might lead to inflation, but one thing is certain. In order to have inflation we must have a potential on moduli space. Furthermore, the natural scale of the potential must be much smaller than the Planck mass (this is the reason why fields other than moduli are not good candidates for the inflaton). Planck scale inflation leads to density fluctuations of order one when it ends. A candidate inflationary universe in such a situation would become highly inhomogeneous and collapse into a collection of black holes on microscopic time scales. A successful slow roll inflationary scenario requires the potential to remain much smaller than the Planck scale while the fields vary by amounts of order the Planck scale.

In dimension higher than 4 there does not appear to be any infrared dynamics which could generate a potential with natural height smaller than the Planck scale. For vacua with a high degree of SUSY, the same arguments which prove their stability show that *no* potential on moduli space is possible. Thus the probability of finding oneself in such a state would be determined by the measure on moduli space, and string theory could be said to predict that it is highly improbable to find the world in a state where more than four dimensions are large.

Note that the key point distinguishing four dimensions here is the ability of marginally relevant four dimensional gauge couplings to produce a large hierarchy of scales. The small potential on moduli space is determined by nonperturbative gauge dynamics. It has often been speculated that this property of four dimensional gauge theories would be an important part of the answer to why we live in four dimensions. Here we have used four dimensional gauge dynamics to construct an explicit cosmological mechanism for explaining this fact. [1]

Vacua with fewer than four dimensions are another question entirely. The above argument does not immediately apply to them. Of particular concern, are two dimensional

[1] It is an interesting open question whether this reasoning also rules out vacua with four large dimensions and extended SUSY. The answer to this has to do with whether such vacua are continuously connected (in configuration space, not along manifolds of classical solutions[2]) to vacua where potentials can be generated. Inflation could then occur in regions where extended SUSY is broken, but the system could settle down into a vacuum with extended SUSY.

vacuum states, because they appear to violate the Bekenstein bound on the entropy of a system of given energy. In D spacetime dimensions, the bound says that the region enclosed inside a $D-2$ surface can have a number of degrees of freedom no larger than the $D-2$ volume of the surface in Planck units. A two dimensional compactification of string theory should be viewed as a ten dimensional spacetime with all but one of the spatial directions compactified at the string scale. If we consider the region to the left of some point in the large direction, it is bounded by a surface of volume the string scale. For any fixed value of the dilaton, Bekenstein's principle would predict that the region had a finite number of states. But string theory appears to predict that the low energy dynamics in this vacuum state is given by a two dimensional field theory, and one would imagine that this implied that there were an infinite number of states to the left of a given point.

In this note, we will show that this expectation is incorrect, and that in fact string theory gives no clear prediction of the number of states. Usually, one establishes the existence of excitations of a given ground state by studying small fluctuations around a classical vacuum configuration, and establishing the spectrum of states in a systematic perturbation expansion. In two dimensions, as a consequence of the infrared divergences of long range fields[2], it is not quite so obvious how to proceed.

We will study the nonlinear classical field equations of the low energy effective field theory. We show that unless the string coupling is exactly zero, arbitrarily small, arbitrarily smooth waves of incoming massless moduli fields lead to naked singularities at which the coupling goes to infinity at finite points of space time. Indeed, these singularities are a consequence of the constraint equations, and follow along with the motion of the massless waves. There is no regular classical solution describing low amplitude smooth coherent states of moduli. We suggest that this is evidence for the nonexistence of the naive spectrum of excitations of two dimensional string vacua. This would remove the apparent contradiction with Bekenstein's principle, and neatly explain why the stringy world is not two dimensional[3].

[2] We note that the vacua in question are Lorentz invariant and have no linear dilaton condensate. Consequently the graviton and dilaton fields are long ranged.

[3] Any Lorentz invariant vacuum state is only an idealization of the situation in the real world.

One possible interpretation of the singularities which we have discovered is the notion of "spontaneous contraction" of two dimensional space time. That is, we interpret the singularity of the classical solution as defining a boundary of space, (making it semi-infinite rather than infinite). The proper distance to the boundary is finite, so if this interpretation is correct, we should expect that there are only a finite number of states between any finite point and the boundary. This would remove the contradiction with Bekenstein's principle. Of course, since the region near the boundary is strongly coupled, we cannot give a convincing argument for this interpretation on the basis of classical field equations. Also note that these boundaries are lightlike, which makes their interpretation somewhat obscure.

In passing, we note also that the *vacuum* state of two dimensional string theory is quite different than that suggested by classical reasoning . Standard arguments suggest that two dimensional string theory has a large moduli space parametrized by a noncompact coset space. It is well known however that nonlinear models in two dimensions have a unique vacuum. In other words, as in the nonlinear quantum mechanics we encountered in higher dimensional cosmology, the ground state wave functional of the model is spread over the entire moduli space. We do not know how to calculate the wave functional, (expecially in string theory where we must also solve two dimensional gravity) but the classical picture of a moduli space of vacuum states is likely to be misleading.

2. Classical Gravity in Two Dimensions

We utilize the notation of Sen's paper[3] to write the equations of two dimensional string theory in conformal gauge as:

$$\partial_+\partial_-(e^{-\Phi}) = 0. \tag{2.1}$$

$$\partial_\pm(ln\lambda)\partial_\pm(e^{-\Phi}) = \partial_\pm^2(e^{-\Phi}) + \frac{1}{4}e^{-\Phi}Tr(\partial_\pm ML\partial_\pm ML). \tag{2.2}$$

The universe has undergone a complicated evolution, and only approximately resembles empty Minkowski space, at late times. A vacuum state with no nearby excitations can never be the endpoint of cosmological evolution.

$$G_{\mu\nu} = \lambda e^{2\Phi} \eta_{\mu\nu}. \tag{2.3}$$

$$x^{\pm} = \frac{1}{\sqrt{2}}(x^0 \pm x^1). \tag{2.4}$$

The moduli are incorporated in the matrix M, which parametrizes a noncompact coset space. We have changed a typographical sign error in Sen's paper. The kinetic terms of the dilaton and moduli fields should have opposite signs. The matrix L is described in [3]. We will not need its detailed form. Indeed, all we really need to use is the fact that the lagrangian of the moduli is conformally invariant, and that the far right hand sides of (2.2)are the left moving and right moving stress tensors of the moduli. It follows that $ln\lambda$ is the sum of a function of x^+ and a function of x^- and can therefore be eliminated by further choice of gauge. The spacetime metric is therefore flat, and the inverse square of the string coupling $e^{-\Phi}$ is the sum of a left moving and right moving function. Let us assume first that we have only left moving stress energy, which is a smooth function of x^+, vanishing at infinity.

Before the left moving pulse arrives, the system is in its vacuum state, and the dilaton is constant. This boundary condition leads to the integral equation:

$$e^{-\Phi} = e^{-\Phi_0} - \int_{-\infty}^{x^+} \int_{-\infty}^{y} dy dz T_{++}(z). \tag{2.5}$$

But now notice that, if the left moving flux is positive, the integral grows without bound as $x^+ \to \infty$. Therefore, unless the initial value of $e^{-\Phi}$ is infinite (*i.e.* the coupling is zero), it will become negative at some finite x^+. This is inconsistent with the reality of the dilaton. Before the negative region is reached, the string coupling goes to infinity.

Thus, in two dimensional string theory, any localized coherent pulse of positive energy matter, no matter how small and smooth, carries with it a singular region in which the semiclassical expansion breaks down. There is no sensible semiclassical picture of excited states of a two dimensional string vacuum. We cannot of course prove that string quantum mechanics does not solve this problem, and restore the naive perturbative picture of an infinite number of states in a region bounded by a finite area, but there is no particular reason to think that this is so.

One possible interpretation of the solutions which we have found, is that of regular spacetimes with a lightlike boundary along the trajectory where the coupling reaches infinity. The proper distance to this boundary is finite, so even in (cutoff) field theory, we expect to be able to associate only a finite number of quantum states with the region between any finite point of space and the boundary[4]. This interpretation thus removes the contradiction with Bekenstein's principle.

Alternatively, it might simply be that the vacuum has *no* excited states. In this view, the classical singularity is a signal of a sickness in the quantum theory. We are not sympathetic to this point of view. We prefer to believe that string theory is a sensible nonsingular theory even in two dimensions. Indeed, as explained in the introduction, one should probably think of all vacuum states of string theory (at least with the same number of supersymmetries) as living in a single connected configuration space. A disease of the two dimensional vacuum should be explained either in the innocuous manner of the previous paragraph or as an instability that drives the system into regions where the low energy two dimensional lagrangian no longer serves as a good approximation to string theory. To examine this question further we will study the two dimensional vacuum state.

2.1. The Two Dimensional Vacuum State

We have argued that there is much less than meets the perturbative eye in the excitation spectrum of two dimensional string vacua. We also believe that the perturbative picture of a moduli space of vacua is substantially modified in the quantum theory. Of course, the enormous duality group of two dimensional string theory already restricts the naive space of vacua to the fundamental domain of the modular group. However, the fundamental domain of the group's action on constant fields is still a multidimensional, noncompact space of finite volume.

In flat spacetime two dimensional field theory, nonlinear models with noncompact target space and negative curvature, are infrared stable. They are well defined as quantum

[4] Let us note here an interesting interpretation (due to E.Witten) of the claim that there are only a finite number of quantum states. Two dimensional string theory has an enormous duality group, which acts on its phase space. Perhaps the fundamental domain of this group action on phase space has finite volume. Then the quantum system would have a finite number of states.

field theories only in the presence of a cutoff. For finite cutoff, such as that provided by string theory, they are interacting quantum mechanical theories. The expansion around a given point in the moduli space is plagued with infrared divergences. Infrared stability implies that the long distance physics is described by a Kosterlitz Thouless phase. That is, the fluctuations around a point are given by logarithmic corrections to a Gaussian model. The long distance observer sees a flat metric on moduli space, since the system is infrared free. Thus, one would imagine that fluctuations tend to drive one into the region of large moduli.

In string theory however these predictions are modified by the effect of two dimensional gravity and by the fact that the region of large moduli is really higher dimensional. We have seen that the gravitational modifications are dramatic for states of finite energy, but we do not know how to compute the gravitational effects on the vacuum. There is however another important modification of the picture outlined above. When the moduli get large, large numbers of light states descend from the string scale. They make the theory higher dimensional (at least in the noncompact parts of moduli space which we understand), and cut off the two dimensional fluctuations. Thus the conclusion that one is driven to large moduli is invalid.

Indeed, it would seem that we have confused the issue by decompactifying the last spatial dimension. If it is compact, we go back to the discussion of the introduction and find that the region of large moduli is not favored. Our discussion of the low energy infrared stable nonlinear model suggests that no potential will be generated in the part of moduli space in which there is one large spatial dimension. Note that this did not depend on the large number of supersymmetries in the problem. The low energy limit of all two dimensional compactifications, whether supersymmetric or not, has a moduli space with negative Ricci curvature.

The dynamics of two dimensional gauge fields, which we have hitherto neglected, does not seem to change this conclusion. Gauge couplings are dimensionful in two spacetime dimensions, and their order of magnitude is given by the string scale. The long distance fluctuations of a two dimensional gauge theory are completely described by a conformally invariant current algebra of gauge invariant currents. This does not generate a potential

on moduli space which could alter the conclusion that large two dimensional volume is improbable.

The most serious loophole in these arguments is our neglect of two dimensional gravitational quantum fluctuations. We have seen that classical gravity dominates the physics of classical states other than the vacuum and completely violates our flat space intuitions. It would be rather remarkable if quantum gravity did not also modify our picture of the vacuum structure. Unfortunately, we do not at present know how to estimate quantum gravitational effects.

To summarize, classical singularities in the long range dilaton field of any matter excitation of low energy two dimensional string theory prevent us from concluding that the theory has the number of states indicated by perturbation theory. The naive picture of a moduli space of vacua is undoubtedly also substantially altered, this time by quantum fluctuations. We have presented an argument that a proper treatment of these fluctuations leads to the conclusion that string theory vacua with one large spatial dimension were about as improbable is those with more than four. However, in high dimension we were justified in ignoring the infrared quantum fluctuations of the gravitational field. This is likely to be incorrect in two dimensions, so our conclusions about the vacuum state must be considered tentative.

2.2. A Short Digression on Three Spacetime Dimensions

It is tempting to try to extend these arguments to three spacetime dimensions, since there too long range graviton and dilaton fields prevent a straightforward perturbative treatment of the theory. We believe however that the situation in three dimensions is quite different. Note that in this case, there is no violation of the Bekenstein bound.

Let us begin by studying the low energy effective heterotic theory and consider classical solutions involving only the gravitational, dilaton, and moduli fields. In the Einstein frame, the lagrangian has the form

$$\sqrt{-g}(R - (\nabla\phi)^2 - G_{ij}(M)(\nabla_i M)(\nabla_j M)). \tag{2.6}$$

Notice that all couplings to the dilaton and other moduli are derivative. Thus, a smooth, small amplitude incoming wave of massless matter will not produce logarithmically growing

dilaton or moduli fields in linear approximation. This is no longer true if we consider solutions involving vector fields, or if we consider corrections to the field equations from higher orders in the string tension expansion. These will lead to nonderivative sources for the dilaton. In linear approximation, the dilaton and other massless fields will grow logarithmically at infinity. One might expect such logarithmically growing scalar fields to lead to infinite energies.

On the other hand, similar logarithmic fields are encountered in the linearized approximation to the static fields of a massive particle. We know however[4] that four dimensional string theory contains stringy cosmic strings with finite mass per unit length. These infinite straight strings have a deficit angle at infinity which is substantial, but less than 2π and seem to be perfectly good, particlelike excitations of three dimensional string theory.

It seems reasonable to conjecture then, that three dimensional string vacua support a variety of states corresponding to finite numbers of moving massive and massless particles. If the particle number gets too large, one of two things happens. For systems (BPS saturated multiparticle states would be one example) in which no bound states are formed, the deficit angle will eventually exceed 2π, and there will be no large spacetime. Alternatively, bound states can be formed of arbitrary numbers of particles with deficit angle less than 2π. Thus, while string dynamics in three spacetime dimensions is highly constrained, and quite odd, there does not seem to be any lack of physical states.

The foregoing discussion applies to weakly coupled string theory. For strong coupling, Witten[5] has suggested that at least some three dimensional string vacua might turn out to look four dimensional. In this case the infrared gravitational effects which make three dimensional physics so bizarre would disappear at strong coupling, and the theory would have many states.

3. Conclusions

The arguments of this paper suggest that classical string vacua with geometries large compared to the string scale have a low probability of being found in the quantum ground state, unless the large spacetime has two, three or four dimensions. Although the two

dimensional case might lead to a satisfactory ground state for string theory, it does not have any nearby excitations. This is a satisfying conclusion because the naive picture of two dimensional string theory appears to violate Bekenstein's entropy bound. There is no longer an obvious counterargument to the claim that string theory is a holographic theory[6].

Note that we do not claim to have solved the dynamics of two dimensional string theory. We merely showed that the heuristic perturbative picture of the spectrum is incorrect. We have also made a preliminary, impressionistic study of the quantum ground state. We argued that the standard picture of a moduli space of vacua was incorrect. On large scales, the two dimensional observer will see a wave function that is spread over moduli space. Considerations of the dynamics of the nonlinear sigma model on moduli space (but neglecting two dimensional gravity, which may be a serious omission) suggest that the large scale wave function tends to be concentrated in noncompact regions of the moduli space. However, precisely in those regions, the effective two dimensional field theory breaks down because the space ceases to be two dimensional. A better approximation to the dynamics is, we believe, obtained by restoring the finite size of the last spatial dimension. Then one might argue that the two dimensional world is similar to that in dimensions higher than 4. The vacuum wave function is concentrated in a region where all spatial dimensions are of order the string scale. Infrared freedom of the sigma model on moduli space implies that no low energy potential for the moduli is generated that could change this conclusion. Furthermore, gauge dynamics in two dimensional string theory leads to confinement on a scale of order the string scale, and should not alter the conclusion either. We are however much less certain of these arguments than we are about the corresponding ones for high dimensions. Two dimensional quantum gravity is not negligible in the infrared, and we have neglected it in our considerations of the ground state. Our classical analysis of states containing arbitrarily small amounts of matter suggests that this may be a serious error.

Although the dynamics of three spacetime dimensions is very odd in string theory, as it will be in any theory containing gravity and massless scalar fields, we could find no reason to expect a restriction on the number of states in the theory. Note that the three dimensional theory does not lead to a violation of Bekenstein's principle. The ultimate

fate of three dimensional vacua of string theory is beyond even our powers of speculation at this point. It may be, as suggested by Witten[5], that some of these vacua are indeed 4 dimensional theories in disguise.

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